

Classical Mechanics — Homework II

Pass 1 for this homework is tentatively due Wednesday September 16, 1998.

- A. Use Lagrange's eq. to obtain the equation of motion for a particle constrained to move (a) vertically and (b) horizontally near the surface of the earth, assuming a dissipation function bv^2 , where b is a constant. In each case, assume an initial velocity v_0 and integrate to find $v(t)$.
- B. Consider a system of mass m with just one generalized coordinate q having a Lagrangian

$$L = e^{bt} \left(\frac{1}{2} m \dot{q}^2 - \frac{1}{2} k q^2 \right)$$

where b and k are each positive constants. Find the equation of motion. What, if any, are the constants of the motion? Describe the possible motion of this system.

- C. Consider a massless spring with spring constant k , fixed at one end, located in a gravitational field \mathbf{g} with a mass m attached to the other end. This is normally called a *spring pendulum*. Let the equilibrium length of the spring without any mass attached be ℓ , and assume that the mass is constrained to move in a vertical plane. Write down the Lagrangian for this system and find the equations of motion. Indicate solutions in the limit of small displacements from equilibrium.
- D. A bead of mass m is threaded on a circular hoop of radius R . The hoop rotates with angular velocity ω about a vertical axis through its diameter. Write down the Lagrangian and find the equations of motion. What is the maximum angular velocity that still allows the bead to maintain a stable equilibrium position?