

Classical Mechanics — Homework III

The final version for this homework is due Wednesday Sep. 30.

- A.** The reflector mirror of a searchlight normally has a parabolic shape (or more precisely, is a paraboloid of revolution). Let z be the axis of revolution. The paraboloid can be described in cylindrical coordinates by $\rho^2 = az$, where a is a constant and ρ is the distance from the z axis to any point on the surface. Suppose that such a mirror points straight up, and a particle of mass m , total energy E , and angular momentum J , slides on it without friction. Use the Lagrange Multiplier method to find the magnitude of the constraint force as a function of ρ and the constants given above.
- B.** Now consider a particle sliding without friction on the surface of a different upward-facing bowl, this time having a spherical shape of radius R . Aside from R , we know only the particle's mass m .
- (1) Determine the Lagrangian in terms of the usual angles θ and ϕ .
 - (2) Determine the generalized momenta p_θ and p_ϕ .
 - (3) Discuss cyclic coordinates and conserved quantities in the context of this example.
 - (4) If $\theta = \theta_0$ (a constant) at all times, find the velocity of the particle.
- C.** Let us go back to the Lagrangian considered in Homework II-B:

$$L = e^{bt} \left(\frac{1}{2} m \dot{q}^2 - \frac{1}{2} k q^2 \right)$$

where b and k are each positive constants. Consider a transformation $s = q \exp(bt/2)$. Rework the problem in terms of the coordinate s . Explain any differences between the solutions in terms of q and s .