PHY 6/75101 October 1998

Classical Mechanics — Homework VI

This homework is due Monday November 2.

A. The piston engine used in a light aircraft of conventional design turns the propeller clockwise as viewed by the pilot. Consider an airplane whose propeller has two blades, each of mass m and length b, and which rotates at a constant angular frequency $\dot{\psi}$. The plane executes a level left turn, and its heading changes at a uniform rate $\dot{\eta}$. What torque vector must be applied to the propeller shaft to maintain this state of motion?

B. A juggler sets a flat plate of mass m and radius R spinning with its center on the tip of one of his fingers. The plate has a uniform angular velocity ω about a vertical axis through its center. The normal to the plane of the plate maintains a constant angle ψ with the vertical. Find the torque vector exerted by the juggler's finger.

C. The three principal axes of a tennis racket are along the handle (moment of inertia I_1), parallel to the strings that are perpendicular to the handle (moment I_2), and perpendicular to the strings and the handle (moment I_3). If the racket is thrown up in the air with a spin about each of the three principal axes in turn, it is much more difficult to catch in the case of the spin about axis #2 than in either of the other two cases. Naive expectations are that axes #2 and #3 should equally easy or difficult, since the rotation of the handle is very similar. In reality, the racket quickly begins to tumble about all three axes when the initial spin is about axis #2, whereas the motion remains much more steady when the racket initially rotates about one of the other two principal axes.

(1) Assuming $I_1 < I_2 < I_3$, use Euler's equations to come up with a quantitative explanation for this observation.

(2) What do you conclude in the case of an object where $I_1 < I_2 = I_3$?

D. Let I_0 be the moment of inertia of a homogeneous cube about an axis passing through the centers of two opposite faces. Find, in terms of I_0, the moment of inertia of the same cube about a space diagonal.

E. Three identical spring pendulums have identical homogeneous solid cubes of mass m and sides ℓ connected at their lower end. One mass is connected to its spring at a corner, another is connected at the center of a face, and the third is connected mid-way along an edge. The springs themselves can be taken as massless. Consider torsional oscillations about the vertical axis of each pendulum, and find the ratio of the periods.