ASTROPHYSICS
Homework Set 1
January 20, 2012

1. The equation for an elliptical orbit with semimajor axis a and eccentricity e is given by
$$r = \frac{a(1-e^2)}{1+e\cos\theta} \; .$$
Show that the corresponding orbit equation in cartesian coordinates has the standard form,
$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1 \; ,$$
and identify the constants x_0, y_0, and b in terms of a and e. (*Hint*: Use the transformation equations $x = r\cos\theta$ and $y = r\sin\theta$.)

2. The equation for an elliptical orbit with semimajor axis a and eccentricity e is given by
$$r = \frac{a(1-e^2)}{1+e\cos\theta} \; .$$
In the limit that $A \gg P$, where A and P are the aphelion and perihelion distances, respectively, the orbit approaches that of a parabola. For a parabolic orbit, $e = 1$.

   (a) Use a limiting procedure to find the orbit equation, $r = r(\theta)$, for a parabolic orbit with perihelion distance P.

   (b) Rewrite your result from part (a) in cartesian coordinates. Express the result in the standard form for a parabola (*e.g.*, $y = a + bx + cx^2$ or $x = \alpha + \beta y + \gamma y^2$).

3. As discussed in class, the orbital velocity for a system of two masses interacting gravitationally can be written as
$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\theta}\left(\frac{dr}{d\theta}\hat{r} + r\hat{\theta}\right) \; ,$$
where $L = mr^2\dot{\theta}$ is the total angular momentum and m is the reduced mass of the system. One can therefore write $\mathbf{v} = v_r\hat{r} + v_\theta\hat{\theta}$, where $v^2 = v_r^2 + v_\theta^2$.

Derive the *vis visa equation*:

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right) \ ,$$

where M is the total mass of the system.

4. A rocketship travels from Earth to Jupiter along the path shown in the diagram. Given that Jupiter has a semimajor axis of 5.2 AU, determine a and e for the orbit of the rocket, and the time required for the rocket to reach Jupiter. (Express a in AU and the trip time in years.)

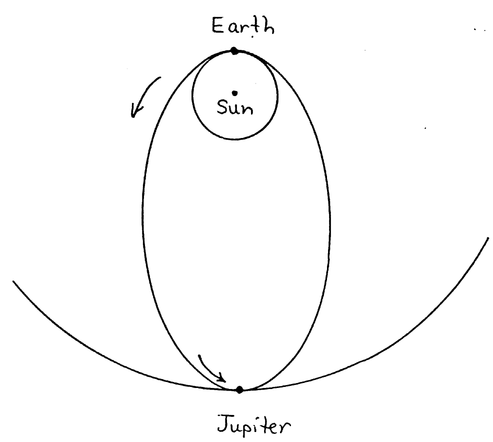

Figure 1: An orbit from Earth to Jupiter.