ASTROPHYSICS
Homework Set 3
February 3, 2012

1. Newton's version of Kepler's third law of planetary motion states that

$$\tau^2 = \frac{4\pi^2}{GM} a^3 \ ,$$

where $G = 6.67 \times 10^{-11}$ N·m^2/kg^2.

   (a) Use the sidereal period of the Moon, $\tau = 27.3$ d, and the Earth-Moon distance of 3.84×10^8 m to determine the mass of the Earth. Express your answer in kilograms (kg).

   (b) Use the Earth-Sun distance of 1.50×10^{11} m to determine the mass of the Sun. Express your answer in kilograms (kg).

2. The surface pressure P_0 for a planet is the ratio of the weight of its atmosphere $m_{\rm atm} g$ to the surface area of the planet $4\pi R^2$, where R is the planet's radius. Use this information to calculate the mass of Earth's atmosphere, $m_{\rm atm}$, in kilograms (kg) and determine the ratio $m_{\rm atm}/M$, where M is the mass of the Earth. *Hint:* for Earth, $P_0 = 1$ atm $= 1.01 \times 10^5$ N/m^2 and $g = 9.8$ m/s^2.

3. Comet Halley has an orbital period of 76 years and an orbital eccentricity of 0.967.

   (a) Determine the comet's perihelion distance and aphelion distance in astronomical units.

   (b) The rotational period of Comet Halley is 54 hours, so it may be considered to be a relatively slow rotator. Determine the subsolar temperature on Comet Halley at perihelion and aphelion. (You can ignore the small albedo effect.)

4. Formaldehyde (H_2CO) has been discovered in interstellar space.

   (a) Calculate $v_{\rm rms}$ for $T = 279$ K. Would our Moon retain this gas for billions of years?

   (b) Calculate $v_{\rm rms}$ and the escape velocity $v_{\rm e}$ for Saturn's satellite Titan. Would Titan retain formaldehyde indefinitely?

5. Tidal friction is slowly increasing the length of the day by about 0.0016 second per century. In the distant future, when the motion of the Earth-Moon system is completely synchronous, the "day" and "month" will become the same and equal to about 47 present days.

   (a) Estimate the distance from the Earth to the Moon as a multiple of its current distance when the motion of the system becomes completely synchronous.

   (b) Assume that the rate at which the length of the day increases remains constant to estimate how long it will take for the Earth-Moon system to become completely synchronous. Express your result in Gy (1 Gigayear (Gy) = 10^9 y). For comparison, note that the Sun is expected to become a Red Giant in about 5 Gy and then (maybe a few hundred thousand years later) will collapse to become a White Dwarf.