CLASSICAL ELECTRODYNAMICS II

Homework Set 1

September 7, 2007

1. Consider transverse electromagnetic waves in free space of the form:

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_0(x,y)\ \mathrm{e}^{i(kz-\omega t)} \ , \\ \mathbf{B} &= \mathbf{B}_0(x,y)\ \mathrm{e}^{i(kz-\omega t)} \ . \end{aligned}$$

   Find the relation between k and ω, as well as the relation between $\mathbf{E}_0$ and $\mathbf{B}_0$. Also show that $\mathbf{E}_0$ and $\mathbf{B}_0$ satisfy the equations for electrostatics and magnetostatics in free space; *i.e.*, show that $\mathbf{E}_0$ and $\mathbf{B}_0$ have no divergence and no curl.

2. Consider the one-dimensional wave equation,

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = 0 \ .$$

   Show that by making the substitutions, $\xi = x - vt$ and $\eta = x + vt$, the wave equation takes on a very simple form that can be easily integrated to give the solution, $u(\xi,\eta) = f(\xi) + g(\eta)$.

3. Consider a source-free medium in which there is no external magnetization and the polarization vector $\mathbf{P}$ is a given function of position and time: $\mathbf{P} = \mathbf{P}(\mathbf{r},t)$. Show that the electric field $\mathbf{E}$ and magnetic induction $\mathbf{B}$ can be written in terms of a single vector $\mathbf{Z}$ (called the Hertz vector) as

$$\mathbf{E} = \nabla \times (\nabla \times \mathbf{Z}) - \frac{\mathbf{P}}{\epsilon} \ , \qquad \mathbf{B} = \mu\epsilon \nabla \times \frac{\partial \mathbf{Z}}{\partial t} \ ,$$

   where with suitable choice of gauge,

$$\nabla^2 \mathbf{Z} - \mu\epsilon \frac{\partial^2 \mathbf{Z}}{\partial t^2} = -\frac{\mathbf{P}}{\epsilon} \ .$$

   How are the usual vector and scalar potentials ($\mathbf{A}$ and Φ) related to $\mathbf{Z}$?