CLASSICAL ELECTRODYNAMICS II

Homework Set 2

September 12, 2008

1. Consider Maxwell's equations when no dielectric or magnetic materials are present. (The equations should be expressed only in terms of the fields $\mathbf{E}$ and $\mathbf{B}$ and the sources ρ and $\mathbf{J}$.)

   (a) Determine the behavior of the sources and fields under the operation of charge conjugation (the act in which the signs of all the source charges are reversed: $q \to q' = -q$).

   (b) Determine the behavior of the sources and fields under the operation of complete spatial inversion ($\mathbf{r} \to \mathbf{r}' = -\mathbf{r}$).

   (c) Determine the behavior of the sources and fields under the operation of time reversal ($t \to t' = -t$).

2. Consider electromagnetic waves in source-free space where $\epsilon = \epsilon_0$ and $\mu = \mu_0$. Given the electric field $\mathbf{E}$ for each part below, calculate the corresponding magnetic induction $\mathbf{B}$ and the Poynting vector, $\mathbf{S} = \mathbf{E} \times \mathbf{B}/\mu_0$.

   (a) $\mathbf{E} = \mathbf{E}_0 \sin(\mathbf{k} \cdot \mathbf{r} - \omega t)$

   (b) $\mathbf{E} = \mathbf{E}_0 \sin(kr - \omega t)$

   (c) $\mathbf{E} = \mathbf{E}_0 \sin(\mathbf{k} \cdot \mathbf{r}) \sin(\omega t)$

   (d) $\mathbf{E} = E_0\, \hat{x} \cos(kz - \omega t) + E_0\, \hat{y} \sin(kz - \omega t)$

   (e) $\mathbf{E} = E_0\, [\hat{x} \cos(\omega t) + \hat{y} \sin(\omega t)] \sin(kz)$

3. Consider a finite volume V that contains both charged particles and electromagnetic fields. Assume that no particles enter or leave V. Show that the Minkowski expressions for the differential and integral forms of the conservation of angular momentum law are

$$\frac{\partial}{\partial t}(\mathcal{L}_{\text{mech}} + \mathcal{L}_{\text{field}}) + \nabla \cdot \overleftrightarrow{\mathbf{M}} = 0$$

and

$$\frac{d}{dt}\int_V (\mathcal{L}_{\text{mech}} + \mathcal{L}_{\text{field}})\, d\tau + \oint_S \overleftrightarrow{\mathbf{M}} \cdot \vec{da} = 0$$

where the field angular-momentum density is

$$\mathcal{L}_{\text{field}} = \mathbf{r} \times \mathbf{g} \ ,$$

with $\mathbf{g} = \mathbf{D} \times \mathbf{B}$ the Minkowski expression for the electromagnetic linear momentum density, and the flux of angular momentum is described by the tensor

$$\overleftrightarrow{\mathbf{M}} = \overleftrightarrow{\mathbf{T}} \times \mathbf{r} \ ,$$

where $\overleftrightarrow{\mathbf{T}}$ is the Maxwell stess tensor.