CLASSICAL ELECTRODYNAMICS II

Homework Set 2

September 25, 2009

1. Consider electromagnetic plane waves propagating in a medium in which $\mathbf{D} = \epsilon\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H}$, where the fields are given by the complex representation:

$$\mathbf{E} = \mathbf{E}_0 \ \mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r}-\omega t)} \ ,$$
$$\mathbf{B} = \mathbf{B}_0 \ \mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r}-\omega t)} \ ,$$

   with $\mathbf{E}_0$ and $\mathbf{B}_0$ complex. Show explicitly that the time-averaged Poynting vector is given by

$$\mathbf{S} = \frac{1}{2}\mathrm{Re}(\mathbf{E} \times \mathbf{H}^*) \ .$$

2. Consider electromagnetic waves in source-free space where $\epsilon = \epsilon_0$ and $\mu = \mu_0$. Given the explicitly real field $\mathbf{E}$ for each part below, calculate the corresponding magnetic induction $\mathbf{B}$, the Poynting vector, $\mathbf{S} = \mathbf{E}\times\mathbf{B}/\mu_0$, and the time-averaged Poynting vector. Interpret each case using, as appropriate, the following descriptors: traveling wave; standing wave; plane wave; spherical wave; linearly polarized wave; circularly polarized wave.

   (a) $\mathbf{E} = \mathbf{E}_0 \sin(\mathbf{k}\cdot\mathbf{r})\sin(\omega t)$

   (b) $\mathbf{E} = E_0 \ \hat{x}\cos(kz-\omega t) + E_0 \ \hat{y}\sin(kz-\omega t)$

3. The most general homogeneous plane wave propagating in the direction $\mathbf{k}$ may be represented as a superposition of two circularly polarized waves:

$$\mathbf{E}(\mathbf{r},t) = (\hat{\epsilon}_+ E_+ + \hat{\epsilon}_- E_-) \ \mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r}-\omega t)} \ ,$$

   where E_+ and E_- are complex amplitudes. Show that if $E_-/E_+ = r \ e^{\mathrm{i}\alpha}$, where r and α are real then the $\mathbf{E}$ vector traces out an ellipse with ratio of semimajor axis to semiminor axis, $(r+1)/(r-1)$, and the axes of the ellipse are rotated by an angle $\alpha/2$ relative to $\hat{\epsilon}_1$ and $\hat{\epsilon}_2$. For convenience, assume that $r > 1$.