NUCLEAR PHYSICS

Homework Set 2

September 22, 2006

1. At distance sufficiently large that overlap between their densities may be ignored, the interaction between two nucleons may be shown to be similar to that between two point dipoles,

$$V(\mathbf{r}) \sim (\sigma_1 \cdot \nabla_1)(\sigma_2 \cdot \nabla_2) f(r) \ .$$

   Under the assumption of one-pion exchange, we may take the radial dependence to have the form

$$f(r) = \frac{\mathrm{e}^{-r/r_0}}{r} \ ,$$

   where

$$r_0 = \frac{\hbar c}{m_\pi c^2}$$

   is the range. The strength of the potential may be related to the pion-nucleon coupling constant g ($g^2/\hbar c \approx 0.081 \pm 0.002$). Except for isospin dependence, which we shall ignore here for simplicity, the potential may be written in the form

$$V(\mathbf{r}) = -g^2 r_0^2 (\sigma_1 \cdot \nabla_1)(\sigma_2 \cdot \nabla_2) \frac{\mathrm{e}^{-r/r_0}}{r} \ .$$

   Show that for $r > 0$, the potential can be written in the form

$$V(\mathbf{r}) = \frac{g^2}{3} \left[\left(1 + \frac{3r_0}{r} + \frac{3r_0^2}{r^2} \right) \ S_{12} + \sigma_1 \cdot \sigma_2 \right] \ \frac{\mathrm{e}^{-r/r_0}}{r} \ ,$$

   where $r = |\mathbf{r}_1 - \mathbf{r}_2|$ and S_{12} is the spin-tensor operator,

$$S_{12} = \frac{3}{r^2} (\sigma_1 \cdot \mathbf{r})(\sigma_2 \cdot \mathbf{r}) - \sigma_1 \cdot \sigma_2 \ .$$

   Hint: Note that $\nabla_2 = -\nabla_1 = \nabla$.

2. The eigenvalues of the spin-tensor operator S_{12} may be obtained by considering the helicity operator

$$\mathcal{H} = \frac{\mathbf{S} \cdot \mathbf{r}}{r} \ ,$$

whose eigenvalues are 0 for $S = 0$, and ± 1 and 0 for $S = 1$. Here, $\mathbf{S} = (\sigma_1 + \sigma_2)/2$ is the total spin operator for the nucleon-nucleon system. Show that $S_{12} = a\mathcal{H}^2 + b\mathbf{S}^2$, where a and b are constants that you should determine. Determine the eigenvalues of S_{12} that correspond to all possible eigenvalues of $\mathcal{H}^2$ and $\mathbf{S}^2$ for the nucleon-nucleon system.