Introduction to Nuclear Physics — Physics 4/56301 — Dr. M.A. Pichowsky
Homework assignment #9 — Clebsch-Gordans: Spin, isospin and parity in decays
Due on April 29 2002

Problem 1: Spin and parity in $N^*(1440)$ decays. There is an interesting particle, referred to as the $N^*(1440)$ because it seems to be an *excitation* of the nucleon N ($N = p$ or n). The spin and isospin of the N^* are $S = 1/2$ and $I = 1/2$, and it has a *positive* intrinsic parity (all, just like the nucleon). The mass $M_{N^*} = 1440$ MeV is somewhat larger than the mass of the nucleon $M_N = 938$ MeV, and therefore the $N^*(1440)$ will decay (via the strong force) into a pion-nucleon state $|\pi N\rangle$.

Note that the eigenvalue of the parity operator $\mathcal{P} = (-1)^L \eta_1 \eta_2$, where η_i is the *intrinsic* parity of the ith particle. (Intrinsic parities may be fundamental or might arise from the quark substructure of the particles in question.) For example, the parity of the pion-nucleon state will be

$$\mathcal{P}|N\pi; LM_L\rangle \;=\; (-1)^L \eta_N \eta_\pi |N\pi; LM_L\rangle \;=\; (-1)^{L+1}|N\pi; LM_L\rangle, \qquad (1)$$

where $\eta_N = +1$ and $\eta_\pi = -1$[1].

a) Consider the decay of a $N^*(1440)$ at rest into a pion and nucleon moving with orbital angular momentum L. The total angular momentum of the intial state is $J = S = 1/2$. The total angular momentum of the final state is $J = L + S$ where $S = S_N + S_\pi$.

Since the strong interaction does not change the total angular momentum of the system, the final and initial angular momenta J are the same. Use this fact (but ignore parity) to find *all* possible values of L for the $N\pi$ state that can result from the N^* decay.

b) Use the fact that parity is conserved in strong interactions to eliminate all possibilities above but one. What value of orbital angular momentum L does the final $N\pi$ system have? (Of course, because we observe that strong decays such as $N^* \to N\pi$ only allow one value of L, we guess that the strong force conserves parity. It is interesting to note that the weak-nuclear force does not conserve parity!)

c) Similarly the N^* resonance also decays into a two-particle state involving a pion and the famous Δ resonance. The Δ is like a nucleon in some ways, but its spin and isospin are $S = 3/2$ and $I = 3/2$.

d) Again, like problem a), give a list of all the possible values of the orbital angular momentum L (between π and Δ) that can arise in the decay $N^* \to \Delta\pi$ (ignoring parity).

e) Now, if you consider parity what values of L are posible?

Problem 2: Isospin in $N^*(1440)$ decays.

a) Lets continue exploring the two decays $N^* \to N\pi$ and $N^* \to \Delta\pi$ by focusing on the isospin properties of these decays.

Please make a table for the four particles $\{N, N^*, \Delta, \pi\}$ involved in the above decays. In the table include the eigenvalues for the isospin I, its projection I_3, and the electromagnetic charges associated with each of the various isospin states[2].

How many charged states are there for each of these particles?

b) Imagine that a positively charged N^* state is produced in a lab, and it then subsequently decays into either $N\pi$ or $\Delta\pi$, as above. Using only isospin considerations, rewrite the state $|N^*; I_{N^*} = \frac{1}{2}, m_{N^*} = +\frac{1}{2}\rangle$ where I_{N^*} is the isospin and m_{N*} is the isospin component along the isotopic-z axis, as a linear combination of the final states $|N\pi; I_N = \frac{1}{2}, m_N, I_\pi = 1, m_\pi\rangle$ for all posible combinations of m_N and m_π (the I_3 eigenvalues for the nucleon and the pion, respectively).

c) Do the same for the positively charged N^* state to decay into all posible charged states of Δ π.

d) How much more often does the decay $N^* \to n\pi^+$ happen over $N^* \to p\pi^0$?[3]

e) How much more often does the decay $N^* \to \Delta^{++}\pi^-$ happen over $N^* \to \Delta^0\pi^+$?[4]

[1]It is useful to remember that the pion is a spin $S = 0$, isospin $I = 1$ particle with negative parity $\eta_\pi = -1$. Typical spin-0 objects are called "scalars" and have positive parity, however, the pion has a negative parity and so it called a "pseudo-scalar."

[2]Clue: The EM charge operator for the N^* and Δ (and for all other baryons) is given by $Q = I_3 + 1/2$, while for π, ρ and ω mesons (and all other mesons) it is $Q = I_3$.

[3]Remember, probabilities are the squares of amplitudes.

[4]If you do these last two questions correctly, you get the experimentally observed ratios! Behold the power of SU(2) isospin!