Classical Mechanics — Homework III

The final version for this homework is due Wednesday Sep. 30.

A. The reflector mirror of a searchlight normally has a parabolic shape (or more precisely, is a paraboloid of revolution). Let \( z \) be the axis of revolution. The paraboloid can be described in cylindrical coordinates by \( \rho^2 = az \), where \( a \) is a constant and \( \rho \) is the distance from the \( z \) axis to any point on the surface. Suppose that such a mirror points straight up, and a particle of mass \( m \), total energy \( E \), and angular momentum \( J \), slides on it without friction. Use the Lagrange Multiplier method to find the magnitude of the constraint force as a function of \( \rho \) and the constants given above.

B. Now consider a particle sliding without friction on the surface of a different upward-facing bowl, this time having a spherical shape of radius \( R \). Aside from \( R \), we know only the particle’s mass \( m \).
   (1) Determine the Lagrangian in terms of the usual angles \( \theta \) and \( \phi \).
   (2) Determine the generalized momenta \( p_\theta \) and \( p_\phi \).
   (3) Discuss cyclic coordinates and conserved quantities in the context of this example.
   (4) If \( \theta = \theta_0 \) (a constant) at all times, find the velocity of the particle.

C. Let us go back to the Lagrangian considered in Homework II-B:

\[
L = e^{bt}(\frac{1}{2}m\dot{q}^2 - \frac{1}{2}kq^2)
\]

where \( b \) and \( k \) are each positive constants. Consider a transformation \( s = q \exp(bt/2) \). Rework the problem in terms of the coordinate \( s \). Explain any differences between the solutions in terms of \( q \) and \( s \).