Classical Mechanics — Homework IV

Pass 1 for this homework is due by 10:00 a.m. Monday October 12.

A. The potential energy of a non-spherical satellite varies slightly, depending on its orientation relative to its orbit. Consider the simplest possible case — a satellite in the form of two identical spherical modules joined by an effectively massless frame. What is the stable orientation of this satellite?

B. A body moves under an attractive central force

\[ f(r) = -\frac{\ell^2}{mr^3} \left( 1 + \frac{d^2}{r^2} \right) \]

where \( \ell \) is angular momentum and \( d \) is a constant. Find and roughly sketch the equivalent one-dimensional potential \( V' \). Show that a spiral path of the form \( r = r_0 + c\theta \) (where \( r_0 \) and \( c \) are constants) leads to this form of force. What can be concluded about the total energy of the body if it is known to follow a spiral path of the above form? Discuss the conditions under which the motion is or is not bounded.

C. A body moves in a spiral path of the (different!) form \( r = c\theta^2 \) under a central force specified by the potential

\[ V(r) = Ar^n + Br^m, \]

where \( A \) and \( B \) are constants. Find the required values of the exponents \( n \) and \( m \).

D. A planet of mass \( m \) orbits a star of mass \( M \). A uniform distribution of dust with density \( \rho \) surrounds the star out to the planet’s orbit.

1. What is the central force?
2. Consider a circular orbit of radius \( R \) with angular momentum \( \ell \). Find a polynomial equation connecting \( R \) and the other parameters given.
3. Next consider small deviations from a circular orbit in the approximation of low dust density. Show that the natural frequency of oscillations is such that the perturbed orbit can be described by a precessing ellipse. Does the precessional angular frequency have the same or opposite sign as the orbital angular frequency?